

## Gravitational, electromagnetic, and transition form factors of the pion\*

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Results of the Spectral Quark Model for the gravitational, electromagnetic, and transition form factors of the pion are discussed. In this model both the parton distribution amplitude and the parton distribution function are flat, in agreement with the transverse lattice calculations at low renormalization scales. The model predictions for the gravitational form factor are compared to the lattice data, with good agreement. We also find a remarkable relation between the three form factors, holding within our model, which besides reproducing the anomaly, provides a relation between radii which is reasonably well fulfilled. Comparison with the CELLO, CLEO, and BaBar data for the transition form factor is also considered. While asymptotically the model goes above the perturbative QCD limit, in qualitative agreement with the BaBar data, it fails to accurately reproduce the data at intermediate momenta.

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The low-energy behavior of the pion is determined by the spontaneous breakdown of the chiral symmetry. This fact allows for modeling the soft matrix elements in a genuinely dynamical way [1–25]. This talk is based on Refs. [26,27] and employs the Spectral Quark Model (SQM) [28] in the analysis of several high-energy processes and their partonic interpretation. This model satisfies *a priori* consistency conditions [28] between open quark lines and closed quark lines, which becomes crucial in the analysis of high-energy processes and enables an unambiguous identification of parton distribution functions and amplitudes. This is not necessarily the feature of other versions of chiral quark models, such as the Nambu–Jona–Lasinio (NJL) model, as was spelled out already in Ref. [1]. For these reasons SQM is particularly well suited for the presented study.

The general theoretical framework is set by the Generalized Parton Distributions (GPDs) [29–37]. These objects arise formally, *e.g.*, from deeply virtual Compton scattering (DVCS) on a hadronic target, effectively opening up the quark lines joining the currents. In local quark models usually the one-loop divergences appear and a regularization is needed. One may either compute the *regularized* DVCS and take the high-energy limit, or compute directly the *regularized* GPD. Besides the requirements of gauge invariance and energy-momentum conservation, this apparently innocuous issue sets a non-trivial consistency condition on admissible regularizations which SQM fulfills satisfactorily.

For the case of the pion, the GPD for the non-singlet channel is defined as

$$\epsilon_{3ab} \mathcal{H}^{q,\text{NS}}(x, \zeta, t) = \int \frac{dz^-}{4\pi} e^{ixp^+ z^-} \langle \pi^b(p') | \bar{\psi}(0) \gamma^+ \psi(z) \tau_3 | \pi^a(p) \rangle \Big|_{z^+=0, z^-=0},$$

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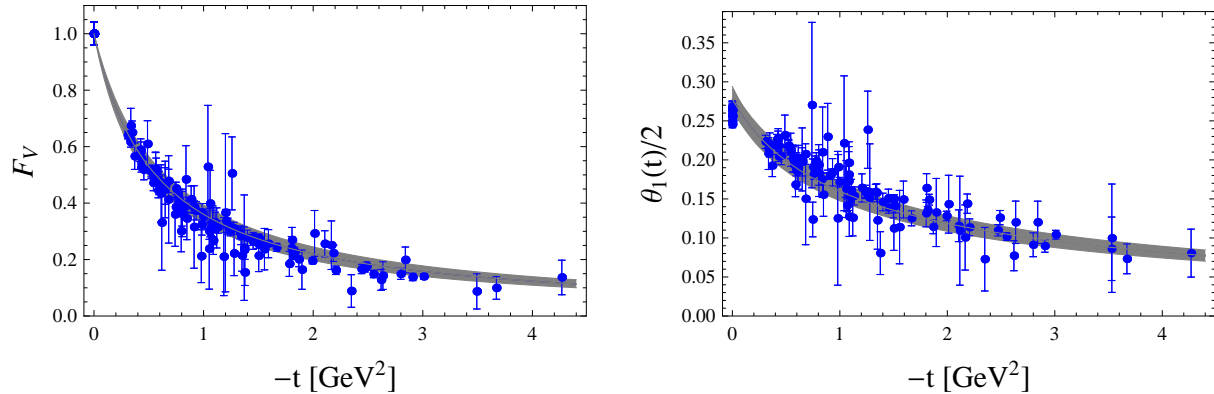


Fig. 1. Form factors of the pion vs. lattice data. Left: the electromagnetic form factor. Right: the quark part of the gravitational form factor,  $\theta_1(t)/2$ , computed in the Spectral Quark Model and compared to the lattice data from Ref. [42]. The band around the model curves indicates the uncertainty in the model parameters.

with similar expressions for the singlet quarks and gluons. We omit the gauge link operators  $[0, z]$ , absent in the light-cone gauge. The kinematics is set by  $p' = p + q$ ,  $p^2 = p'^2 = m_\pi^2$ ,  $q^2 = -2p \cdot q = t$ . The variable  $\zeta = q^+/p^+$  denotes the momentum fraction transferred along the light cone. Formal properties of GPDs can be elegantly written in the symmetric notation involving the variables  $\xi = \frac{\zeta}{2-\zeta}$ ,  $X = \frac{x-\zeta/2}{1-\zeta/2}$ :

$$H^{I=0}(X, \xi, t) = -H^{I=0}(-X, \xi, t), \quad H^{I=1}(X, \xi, t) = H^{I=1}(-X, \xi, t).$$

For  $X \geq 0$  one has  $\mathcal{H}^{I=0,1}(X, 0, 0) = q^{S,NS}(X)$ , where  $q(x)^i$  are the standard parton distribution functions (PDFs). In QCD all these objects are subjected to radiative corrections, as they carry anomalous dimensions, and become scale-dependent, *i.e.* they undergo a suitable QCD evolution. This raises an important question: what is the scale  $Q_0$  of the quark model when matching to QCD is performed? The momentum-fraction sum rule fixes this scale to be admittedly very low,  $Q_0 = 313_{-10}^{+20}$  MeV, for  $\Lambda_{\text{QCD}} = 226$  MeV. Remarkably, but also perhaps unexpectedly, this choice, followed by the leading-order evolution, provides a rather impressive agreement with the high energy data, as well as the Euclidean and transverse-lattice simulations (see Ref. [26] for a detailed summary).

The following *sum rules* hold for the moments of the GPDs:

$$\int_{-1}^1 dX H^{I=1}(X, \xi, t) = 2F_V(t), \quad \int_{-1}^1 dX X H^{I=0}(X, \xi, t) = 2\theta_2(t) - 2\xi^2\theta_1(t),$$

where  $F_V(t)$  denotes the *vector form factor*, while  $\theta_1(t)$  and  $\theta_2(t)$  stand for the *gravitational form factors* [38]. Other important features are the *polynomiality* conditions [29], the *positivity bounds* [39, 40], and a low-energy theorem [41]. We stress that all these properties required on formal grounds are satisfied in our quark-model calculation [26]. Unlike GPDs, the form factors of conserved currents do not undergo the QCD evolution.

In the chiral limit we have the following identity in SQM relating the gravitational and electromagnetic form factor,

$$\frac{d}{dt} [t \theta_i(t)] = F_V(t), \quad (i = 1, 2), \quad (1)$$

from which the identity between the two gravitational form factors  $\theta_1(t) = \theta_2(t) \equiv \Theta(t)$  follows.

Since there is no data for the full kinematic range for the GPDs of the pion, we present here the results for the generalized form factors only, in particular for the gravitational ones. It is

well known that the data for the electromagnetic form factor are well parameterized with the monopole form, which by construction is reproduced in SQM, where the vector meson dominance is built in. The gravitational form factors are available from the lattice QCD simulations [42,43]. In Fig. 1 the electromagnetic form factor and the quark part of the gravitational form factor are compared to the lattice data. We note a very good agreement. In SQM one has the relation

$$m_\rho^2 = 24\pi^2 f^2 / N_c, \quad (2)$$

where  $f$  is the pion weak decay constant in the chiral limit. This relation works within a few percent phenomenologically. The expressions for the form factors in SQM are very simple,

$$F_V(t) = \frac{m_\rho^2}{m_\rho^2 - t}, \quad \theta_{1,2}(t)/\theta_{1,2}(0) = \frac{m_\rho^2}{t} \log \left( \frac{m_\rho^2}{m_\rho^2 - t} \right). \quad (3)$$

We note the longer tail of the gravitational form factor in the momentum space, meaning a more compact distribution of energy-momentum in the coordinate space. Explicitly, we find a quark-model formula

$$2\langle r^2 \rangle_\theta = \langle r^2 \rangle_V. \quad (4)$$

The two previous processes regard two pions and either one photon or one graviton in the corresponding three-point vertex function. An apparently disparate object is given by the pion-photon *transition distribution amplitude* (TDA) [44,45]

$$\int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle \gamma(p', \varepsilon) | \bar{\psi}(0) \gamma^\mu \frac{\tau^a}{2} \psi(z) | \pi^b(p) \rangle \Big|_{z_T=0}^{z_+=0} = \frac{ie}{p^+ f} \epsilon^{\mu\nu\alpha\beta} \varepsilon_\nu p_\alpha q_\beta V^{ab}(x, \zeta, t), \quad (5)$$

Here the photon carries momentum  $p' = p + q$  and has polarization  $\varepsilon$ . As before, the presence of the gauge link operators is understood in Eq. (5) to guarantee the gauge invariance of the bilocal operators. We consider here the *isovector* quark bilinears. Since the photon couples to the quark through a combination of the isoscalar and isovector couplings, *i.e.* the quark charge is  $Q = 1/(2N_c) + \tau^3/2$ , one has the isospin decomposition

$$V^{ab}(x, \zeta, t) = \delta^{ab} V_{I=0}(x, \zeta, t) + i\epsilon^{abc} V_{I=1}(x, \zeta, t). \quad (6)$$

The isoscalar form factor is related to the pion-photon *transition form factor* by the sum rule

$$F_{\pi\gamma\gamma^*}(t) = \frac{2}{f} \int dx V^{I=0}(x, \zeta, t), \quad (7)$$

where the factor of 2 comes from the fact, that either of the photons can be isoscalar. The form factor in SQM was obtained directly in Ref. [28] and later on from the integration of the pion-photon isoscalar transition distribution amplitude (TDA) yielding [21] a  $\zeta$ -independent function (as required by polynomiality),

$$F_{\pi\gamma\gamma}(t, A) = \frac{2f}{N_c} \left[ \frac{2m_\rho^2}{m_\rho^4 - tm_\rho^2 + (1-A^2)t^2} + \frac{1}{At} \log \left( \frac{2m_\rho^2 - (1-A)t}{2m_\rho^2 - (1+A)t} \right) \right], \quad (8)$$

where  $A = (q_1^2 - q_2^2)/(q_1^2 + q_2^2)$  is the photon asymmetry parameter. For  $A = 1$  we have

$$F_{\pi\gamma\gamma^*}(t) = \frac{1}{12\pi^2 f} \left[ \frac{2m_\rho^2}{m_\rho^2 - t} + \frac{m_\rho^2}{t} \log \left( \frac{m_\rho^2}{m_\rho^2 - t} \right) \right], \quad (9)$$

where relation (2) has been used. We read out from this formula the corresponding rms radius to be  $\langle r^2 \rangle_{\pi\gamma\gamma^*}^{1/2} = \sqrt{5}/m_\rho = 0.57$  fm for  $m_\rho = 770$  MeV. Equivalently, one may use the slope

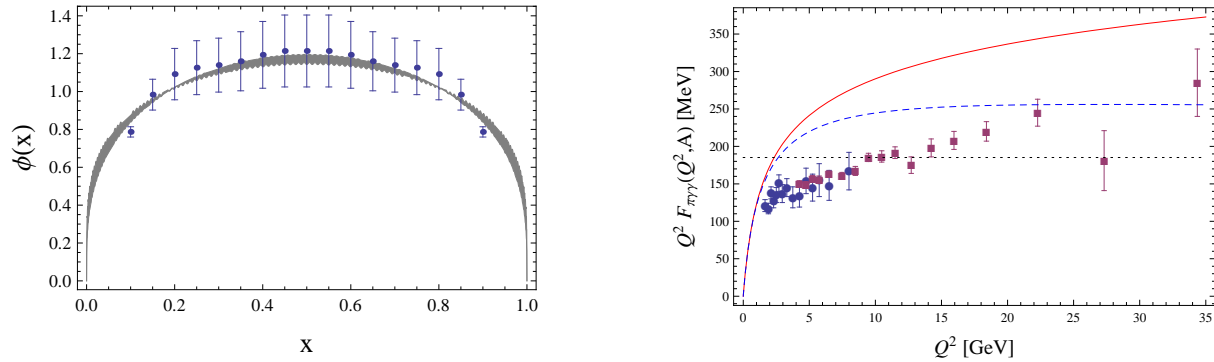


Fig. 2. Left: chiral quark model prediction for the pion DA evolved to the scale of  $0.5\text{GeV}$  (band) and compared to the transverse lattice data [54]. Right: the pion transition form factor compared to the CLEO [47] and BaBar [48] data. Solid (dashed) lines are the SQM prediction at  $A = 1$  ( $A = 0.95$ ). The dotted line is the perturbative QCD prediction.

parameter  $b_\pi = \frac{d}{dt} F_{\pi^0 \gamma \gamma^*}(t) / F_{\pi^0 \gamma \gamma^*}(t) \Big|_{t=0}$ . SQM gives  $b_\pi = 5/(6m_\rho^2) = 1.4 \text{ GeV}^{-2}$ , in a very reasonable agreement with the experimental value  $b_\pi = (1.79 \pm 0.14 \pm 14) \text{ GeV}^{-2}$ , originally reported by CELLO [46]. A comparison of Eq. (8,9) to the CLEO [47] and BaBar [48] data is presented in the right panel of Fig. 2. The solid line corresponds to the model calculation with  $A = 1$ , while the dashed line is for  $A = 0.95$ . We note that the experiment does not produce strictly real photons, thus the observed sensitivity to the value of  $A$  is a relevant effect. We note that while at  $|A| = 1$  the model asymptotics for the transition form factor is  $(2f/N_c) \log(-t/m_\rho^2)/(-t)$ , at  $|A| \neq 1$  it becomes  $(2f/N_c) \log[(1+A)/(1-A)]/(-At)$ . The behavior is clearly seen in Fig. 2. As we notice, in the intermediate range of  $Q$  SQM overshoots the data.

The recent BaBar measurements [48] have predated the long-standing perturbative QCD prediction [49, 50] that  $-tF_{\pi\gamma\gamma^*}(t)$  goes asymptotically to a constant value of  $2f$ . Some authors [51, 52] have pointed out that the key to this unexpected behavior hints for a flat pion PDA and the end-point singularities and switched-off QCD evolution. The flatness of the PDA at low renormalization scales has been originally found in the Nambu–Jona-Lasinio model [10] and in SQM [28].

We note in passing that a constant PDA is also found in the Regge model [53].

Remarkably, an almost flat PDA is also found non-perturbatively on the transverse lattice [54] (see the left panel of Fig. 2). Actually, the non-vanishing of the PDA at the end points (at the quark-model scale) is not only a consequence of local quark models. Nonlocal models correctly implementing the chiral Ward-Takahashi identity also get such a feature [18]. A trend to flatness is observed in contrast to calculations violating the chiral symmetry constraints. However, the corresponding transition form factor in non-local models does not show a steep rise [55] as suggested by the BaBar data. The calculation in Ref. [56, 57], which reproduces the CLEO and BaBar data, requires, unfortunately, a much too small constituent quark mass, which is incompatible with other sectors of the pion phenomenology. The apparent inconsistency of the BaBar data with the QCD convolution scheme is also addressed in Ref. [58, 59].

Let us remind the reader that according to the conventional perturbative QCD approach, the radiative corrections are computed order by order in the twist expansion. Most often this is in practice possible only for the leading-twist contribution. Actually, this is the only way to identify the PDA within a non-perturbative scenario or quark model calculations. In fact, the chiral quark models require a low scale not only by fixing the second Gegenbauer coefficient  $a_2$  of the PDA. As already mentioned, the same conclusion is reached independently by fixing the momentum fraction of the valence quarks to its natural 100% value at the quark-model scale, where the quarks constitute the only degrees of freedom.

On a more methodological level, it is worth mentioning that the conventional NJL model does not share some of the virtues of SQM, particularly the interplay between chiral anomaly and factorization, a subtle point which was discussed at length in Ref. [11] for the NJL case. The  $\pi\gamma\gamma$  triangle graph is linearly divergent, and thus a regularization must generally be introduced. If one insists on preserving the vector gauge invariance, the regulator must preserve that symmetry, but then the axial current is not conserved, generating the standard chiral anomaly. The obvious question arises whether the limit  $Q^2 \rightarrow \infty$  must be taken before or after removing the cut-off. If one takes the sequence  $Q^2 > \Lambda^2$ , a constant PDA is obtained in agreement with our low energy calculation. For the opposite sequence factorization does not hold in NJL. The good feature of SQM is that the spectral regularization does not make any difference between the two ways. This illustrates in a particular case the above-mentioned general consistency requirement between regularized open and closed quark lines (see e.g. [60]).

Finally, by combining Eq. (3) and Eq. (9) we get the remarkable relation among the electromagnetic, gravitational and transition form factors, holding in SQM:

$$F_{\pi\gamma\gamma^*}(t) = \frac{1}{12\pi^2 f} [2F_V(t) + \Theta(t)] , \quad (10)$$

whence

$$3\langle r^2 \rangle_{\pi\gamma\gamma^*} = 2\langle r^2 \rangle_V + \langle r^2 \rangle_\Theta . \quad (11)$$

The previous relation is not fulfilled in the conventional NJL model. Of course, it would be interesting to test the relation Eq. (10) against the future data or lattice QCD.

In conclusion, we note that while the description of the pion transition form factor in a genuinely dynamical way remains a challenge, the Spectral Quark Model offers many attractive features which are required from theoretical consistency. It satisfies the chiral anomaly and the factorization property. The vector and gravitational form factors describe experimental and/or lattice-QCD data satisfactorily. A remarkable model relation among the gravitational, electromagnetic and transition form factors has also been deduced. Finally, for the latter, we have also displayed a hitherto unnoticed sensitivity to the photon momentum asymmetry parameter  $A$  which might be relevant for other studies.

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